JOURNAL OF MATERIALS SCIENCE 16 (1981) 3346–3356

Prediction of the thermal conductivity of insulation materials

D. M. DAWSON, A. BRIGGS
Materials Development Division, AERE, Harwell, Oxon, UK

From a theoretical and experimental examination of thermal conductivity expressions proposed in the literature, a geometric mean equation is found to give predictions which most closely agree with experimental data for debased alumina materials with a range of porosities. Using the equation a computer program is developed which predicts conductivities of multi-component systems and presents data in the form of a three-component diagram with iso-conductivity lines.

1. Introduction

As energy costs increase, and become a more significant proportion of the total cost of processes and materials, an increased awareness of the need for efficient thermal insulation is developing. In addition to the application of insulation in previously unconsidered parts of processes, there is also a move towards greater insulation efficiency in areas where insulation has traditionally been used. Consequently, in the former case materials may be subjected to new and possibly more severe conditions and in the latter case more demanding constraints are placed on their design. To satisfy these more stringent requirements, new materials, or new formulations of existing materials, are necessary. The properties required of suitable materials, such as low conductivity, adequate strength and thermal stability, are often incompatible, and a compromise must be sought which demands a composite materials approach. There is then a need for optimization of the designs of materials containing a number of components. If a simple method for predicting the conductivity of such materials were available, much fruitless experimental work could be avoided.

Prediction of the thermal conductivities of multicomponent materials has been an aim of workers in the field for many years, and yet the construction of a model which accurately represents real materials has proved to be difficult. This has been due, not to ignorance of the laws or mechanisms of heat transmission involved, but to the intractability of their application to sufficiently realistic models. Many relationships based on experimental data, or theoretical models with various degrees of realism have been proposed [1–26]. The purpose of the present paper is to select from these available models an expression of general applicability, which demonstrably gives reasonable agreement with experimental results, and can be used as a basis for the development of a computer program for the prediction of the thermal conductivities of multicomponent materials.

2. Assessment of the mathematical models

The approaches that have been used to predict or describe the thermal conductivities of materials containing more than one component can be classified in three basic groups:

(a) Flux-law models [1–9];
(b) Ohm's-law models [10–21];
(c) Empirical relationships [22–27].

Maxwell [1] developed a relationship between the electrical resistance of a material, its component resistances and their volume fractions using a flux-law method. The model considered spheres distributed in a matrix so that their electrical fields could be said to be non-interactive, that is, that their separation distances were large compared to their radii, and, therefore, the volume fraction of the spheres was small. Although Maxwell's work [1] dealt exclusively with the electrical resistance of a composite it is equally applicable in the field of thermal conductivity, and has been quoted by many workers (e.g. [12, 27]) in that field.

0022–2461/81/123346–11$03.10/0

Generally the flux-law approach derives the temperature gradient of one of the following model systems:

(i) A regular array of spherical particles which may be in contact or dispersed.

(ii) A random distribution of spheres dispersed in a matrix phase so that no interaction between the temperature fields of the particles occurs.

Using these temperature gradients the effective thermal conductivity is established from the fundamental heat-conduction law, attributed to Fourier, which states that the heat flux is proportional to the temperature gradient

$$q = -k\nabla T, \tag{1}$$

where q is the heat flux, k is the proportionality constant, ∇ is the thermal conductivity and T is the sum of the temperature gradient components along the Cartesian axes of the model. For one-dimensional heat transfer, therefore, Equation 1 reduces to

$$q = -k\frac{\mathrm{d}T}{\mathrm{d}x}, \tag{2}$$

where x is distance.

Equations 1 and 2 indicate the simplicity of the application of the flux law in obtaining the thermal conductivity of a system. A major problem, however, is the determination of the temperature field inside a body. In the cases of the sphere arrangements described above, this problem is not insurmountable, but for models which are more representative of real systems the solutions are complex. As a result the flux-law models developed are of relatively simple geometry and bear little resemblance to real multicomponent materials.

This major draw-back gave rise to a greater interest in the Ohm's-law models which are derived from an electrical analogue of a system of resistances, assuming one-dimensional heat transfer through a unit cell of material and, over the years, these models have been developed with increasing realism. Russell [10] developed one of the early model systems using the electrical analogy. He considered two extreme cases as shown in Fig. 1a and b.

Using these two model systems Russell [10] derived two equations giving the relationship between the composite conductivity and the volume fractions and conductivities of the two components. For a continuous solid phase

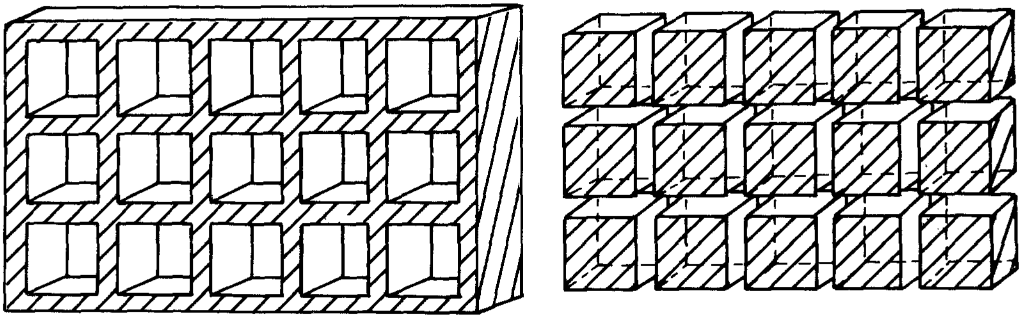
$$k_\mathrm{m} = k_\mathrm{s}\left[\frac{\phi^{2/3} + \frac{k_\mathrm{s}}{k_\mathrm{p}}(1-\phi^{2/3})}{\phi^{2/3} - \phi + \frac{k_\mathrm{s}}{k_\mathrm{p}}(1-\phi^{2/3}+\phi)}\right] \tag{3}$$

and for a continuous porous phase

$$k_\mathrm{m} = k_\mathrm{p}\left[\frac{(1-\phi)^{2/3} + \frac{k_\mathrm{p}}{k_\mathrm{s}}(1-(1-\phi)^{2/3})}{\phi + (1-\phi)^{2/3} - 1 + \frac{k_\mathrm{p}}{k_\mathrm{s}}(2-\phi-(1-\phi)^{2/3}}\right], \tag{4}$$

where k_m is the composite conductivity, k_s is the solid-phase conductivity, k_p is the porous-phase conductivity and ϕ is the volume fraction of the porous phase.

It would be expected that a real system would be better described by a combination of both of these models, and would hence exhibit a conduc-



(a) (b)

Figure 1 The models proposed by Russell [10] to aid in the calculation of the thermal conductivity of a porous system. (a) shows a continuous split phase with isolated cubic pores. (b) shows a continuous porous phase with discontinuous cubic solid particles.

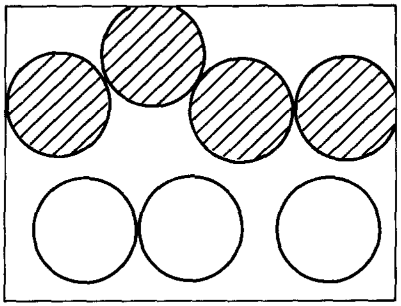


Figure 2 The Wyllie and Southwick [11] model of a conducting network of spheres in good electrical contact (shaded) and in poor or non-existent contact (unshaded) within a conducting matrix electrolyte.

tivity somewhere between the calculated values of Equations 3 and 4. A model which combines all the possible elemental arrangements of the phases is therefore required.

Wyllie and Southwick [11] developed a three-element resistor model for calculating the electrical conductivity of an aggregate of conducting particles in a conducting electrolyte. These three elements were considered to be:

(a) particles and electrolyte in series;

(b) particles in close contact, thus forming a continuous solid conduction path;

(c) the electrolyte filling the interstices.

This model is depicted in Fig. 2. In this, and all subsequent model diagrams the electrical or thermal flux will be from left to right.

The model in Fig. 2 can be simplified to give a unit cell of the three parallel resistances described above, as shown in Fig. 3.

Using this model Wyllie and Southwick [11] derived an equation which is a sum of weighted component elements:

$$k_\mathrm{m} = \frac{k_\mathrm{s}k_\mathrm{p}}{xk_\mathrm{s} + yk_\mathrm{p}} + \frac{k_\mathrm{p}}{F} + \frac{k_\mathrm{s}}{Z}, \tag{5}$$

where the definitions given [11] for x and y are that they are dimensionless parameters descriptive of the particle arrangement and the pore arrangement in the series component, Z is a dimensionless parameter descriptive of the particles in contact and F is the "formation factor" of the particles. An examination of Equation 5 and Fig. 3 indicates that

$$x + \frac{1}{F} = \text{pore volume-fraction} \tag{6}$$

and

$$y + \frac{1}{Z} = \text{solid volume-fraction.} \tag{7}$$

A problem in using Equation 5 is deciding the proportions of series and parallel components of solid and porous phases.

Woodside and Messmer [12] adapted this model for the calculation of the thermal conductivities of two-phase systems and developed it to give a weighted mean equation of the form

$$k_\mathrm{m} = \frac{ak_\mathrm{s}k_\mathrm{p}}{k_\mathrm{s}(1-d) + k_\mathrm{p}d} + bk_\mathrm{s} + ck_\mathrm{p}, \tag{8}$$

where the terms a, b, c and d are defined in Fig. 4.

Dul'nev and Zarichnyak [14] derived an expression similar to Equation 8 with numerical values for the terms a to d

$$k_\mathrm{m} = \frac{\frac{2}{3}k_\mathrm{s}k_\mathrm{p}}{k_\mathrm{s}\phi + k_\mathrm{p}(1-\phi)} + \frac{1}{3}(k_\mathrm{s}(1-\phi) + k_\mathrm{p}\phi). \tag{9}$$

This expression is also attributed to Schuhmeister [28] who worked on the conductivity of textile fabrics.

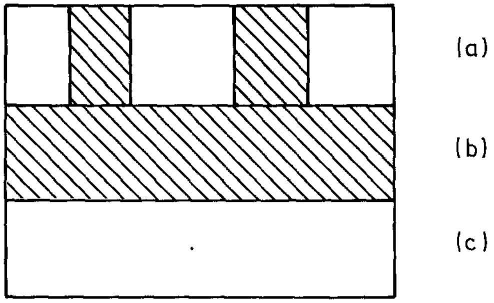


Figure 3 The Wyllie and Southwick [11] three-element resistor model. The letters refer to the text above (solid-phase shaded).

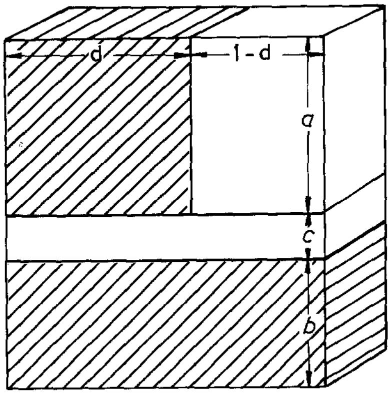


Figure 4 The Woodside and Messmer three-element resistor model. The letters refer to the text above (solid-phase shaded).

Dul'nev [16] derived an equation for a unit cell of different configuration consisting of interpenetrating components of the form

$$k_{\mathrm{m}} = k_{\mathrm{s}}\left[c^2 + \frac{k_{\mathrm{p}}}{k_{\mathrm{s}}}(1-c)^2 + \frac{2\dfrac{k_{\mathrm{p}}}{k_{\mathrm{s}}}c(1-c)}{\dfrac{k_{\mathrm{p}}}{k_{\mathrm{s}}}c + (1-c)}\right], \tag{10}$$

where $c = \Delta/L$ and Δ and L are defined in Fig. 5.

For one-dimensional thermal conduction the model depicted in Fig. 5 reduces to the same unit cell as Fig. 4, and so, for the purpose of this work there is no advantage in using Equation 10 in preference to Equations 8 or 9.

An alternative approach to Fig. 4 is to assume that the series and parallel components are arranged in a random fashion which, for a material consisting of particles which are small compared with the sample size, is not unreasonable. Given this assumption a geometric mean equation can be derived such that

$$k_{\mathrm{m}} = \Pi k_i^{(V_{\mathrm{f}})_i}, \tag{11}$$

where k_i and V_{f_i} represent, respectively, the thermal conductivities and volume fractions of the i components.

This equation is quoted by Wimmer *et al.* [29] in its logarithmic form and is also given in the two-component form

$$k_{\mathrm{m}} = k_{\mathrm{s}}^{(1-\phi)}k_{\mathrm{p}}^{\phi} \tag{12}$$

by Woodside and Messmer [12].

There are many empirical relationships between the effective conductivity of a material and the volume fractions of its components and their conductivities. A large number of these relationships take an equation derived from a model and include an empirical value to give the best fit to the available data. Chaudhary and co-workers [17–20] used a variation on the equation derived from the model shown in Fig. 4, which takes the form:

$$k_{\mathrm{m}} = (dk_{\mathrm{p}} + bk_{\mathrm{s}})^n\left[\frac{k_{\mathrm{p}}k_{\mathrm{s}}}{ck_{\mathrm{s}} + ak_{\mathrm{p}}}\right]^{1-n}, \tag{13}$$

where

$$n = \frac{\log \phi\dfrac{k_{\mathrm{m}}}{k_{\mathrm{p}}} + (1-\phi)\dfrac{k_{\mathrm{m}}}{k_{\mathrm{s}}}}{\log\left[1 - \phi(1-\phi)\left(\dfrac{k_{\mathrm{s}}}{k_{\mathrm{p}}} + \dfrac{k_{\mathrm{p}}}{k_{\mathrm{s}}} - 2\right)\right]}. \tag{14}$$

Over the series of papers [17–20] n was simplified to [20]

$$n = \frac{0.5\,(1-\log\phi)}{\log\left(\phi(1-\phi)k_{\mathrm{s}}/k_{\mathrm{p}}\right)}. \tag{15}$$

In the same work Chaudhary and Bhandari [20] gave an alternative to Equation 13 which was again of the same form as Equations 8 and 9

$$k_{\mathrm{m}} = (\phi k_{\mathrm{p}} + (1-\phi)k_{\mathrm{s}})^n\left(\frac{\phi}{k_{\mathrm{p}}} + \frac{(1-\phi)}{k_{\mathrm{s}}}\right)^{n-1}, \tag{16}$$

where n is as before.

Empirical expressions in the form of the geometric mean equation for two components were also proposed by Lichtenecker [24] and Assad [25]. The Lichtenecker equation is identical to Equation 12 but Assad added an empirical factor to give:

$$k_{\mathrm{m}} = k_{\mathrm{s}}\left(\frac{k_{\mathrm{p}}}{k_{\mathrm{s}}}\right)^{\phi c}, \tag{17}$$

where c is said to be approximately unity.

The above equations by no means exhaust the list of those proposed in the past, but are considered to be broadly representative of the general categories investigated.

A limitation of all the equations is that they take account only of the conductivity of the system that arises through phonon transmission and gas conduction; no factors for radiative or convective heat transfer have been included. Furthermore, variations of phonon and gas conduction that occur with temperature change are not accounted for by these models, so the equations are effectively iso-thermal. As such

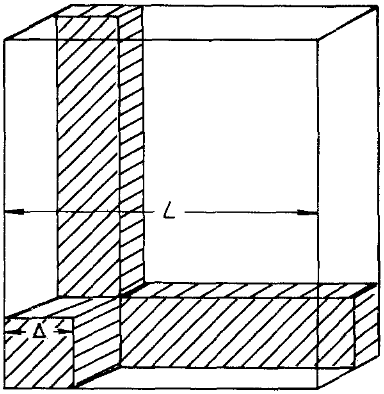


Figure 5 The Dul'nev [16] unit-cell model for two-component thermal conductivity calculations.

their application is limited to situations in which radiative and convective heat transfer mechanisms are either not operating or are insignificant when compared to phonon or gas conductivities. In the latter case the use of experimental data, which, being "effective" conductivities, necessarily account for all the heat transfer mechanisms operating at a given temperature, is permissible.

In addition to the temperature limitation on the use of the geometric mean equations mentioned above, there are other constraints on their use which must be taken into account:

(a) the arrangement of the component phases should be random;

(b) their particle sizes should be small in relation to the sample size;

(c) the components should form a simple physical mixture; and

(d) the conductivity and volume-fraction data must be available for the components in the same condition as they occur in the composite.

However, as many materials fulfil these conditions approximately, in the majority of cases, the above limitations are not serious.

TABLE I A compilation of the equations tested and their validity over the whole range of compositions

Equation	Validity	Equation number
$k_m = k_s \dfrac{\phi^{2/3} + \dfrac{k_s}{k_p}(1 - \phi^{2/3})}{\phi^{2/3} - \phi + \dfrac{k_s}{k_p}(1 - \phi^{2/3} + \phi)}$	Valid	(3)
$k_m = k_p \dfrac{(1 - \phi)^{2/3} + \dfrac{k_p}{k_s}(1 - (1 - \phi)^{2/3})}{\phi + (1 - \phi)^{2/3} - 1 + \dfrac{k_p}{k_s}(2 - \phi - (1 - \phi)^{2/3}}$	Valid	(4)
$k_m = \dfrac{k_s k_p}{x k_s + y k_p} + \dfrac{k_p}{F} + \dfrac{k_s}{Z}$	*	(5)
$k_m = \dfrac{a k_s k_p}{k_s(1 - d) + k_p d} + b k_s + c k_p$	Valid	(8)
$k_m = \dfrac{\frac{2}{3} k_s k_p}{k_s \phi + k_p(1 - \phi)} + \dfrac{1}{3}(k_s(1 - \phi) + k_p \phi)$	Valid	(9)
$k_m = k_s \left[c^2 + \dfrac{k_p}{k_s}(1 - c)^2 + \dfrac{2 \dfrac{k_p}{k_s} c(1 - c)}{\dfrac{k_p}{k_s} c + (1 - c)} \right]$	Valid	(10)
$k_m = \Pi k_i^{(V_f)_i}$	Valid	(11)
$k_m = k_s^{(1-\phi)} x k_p^{\phi}$	Valid	(12)
$k_m = (d k_p + b k_s)^n \left[\dfrac{k_p k_s}{c k_s + a k_p} \right]^{1-n}$	*	(13)
$k_m = (\phi k_p + (1 - \phi) k_s)^n \left(\dfrac{\phi}{k_p} + \dfrac{(1 - \phi)}{k_s} \right)^{n-1}$	Valid	(16)
$k_m = \left[k_s \dfrac{k_p}{k_s} \right]^{\phi c}$	†	(17)

*These equations cannot be checked by this method because of the presence of several unknown parameters. It must be noted however that both are of the same form as other equations which have been proved to be valid.

†The Assad [25] equation will be valid only if the value for c is unity for the two extreme volume fractions.

3. Evaluation of conductivity equations

Each equation discussed in Section 2 will be considered in two ways:

(a) Is it valid for the whole range of compositions in any two-phase system?

(b) How do the values calculated from the equation compare with experimentally-obtained values for a two-phase test system?

Each equation was examined at the two extremes of $\phi = 0$ and $\phi = 1$. In the former case, when $\phi = 0$, the material is pure solid, and hence $k_m = k_s$, and in the latter, when $\phi = 1$, then $k_m = k_p$. Table I shows the results of the evaluation.

The majority of the equations listed in Table I are valid, and those for which the check could not be carried out may also be valid when empirical values are substituted for the unknown values. Since it is a main aim of the present work to select an expression that can be used to predict a conductivity for any material or proposed material, an equation is required which is valid for any materials system, and preferably has no empirical variable. This requirement eliminates Equations 5, 8, 10, 13, 16 and 17 from consideration, even though they have been shown to be valid for all volume fractions of solid material and porosity. The remaining equations, Equations 3, 4, 9, 11 and 12, are valid at volume-fraction extremes, require no empirical factors in order to calculate conductivities, and so may be suitable for our purpose.

If curves are produced for two-component systems using Equations 3, 4, 9, 11 and 12 it can be seen (Fig. 6) that, at least for small component conductivity ratios, the choice of equation from which to calculate the conductivity of a material is unimportant since the lines are quite closely grouped. However, when there are large differences in the conductivities of the two components, the curves exhibit marked divergence, as seen in Fig. 7.

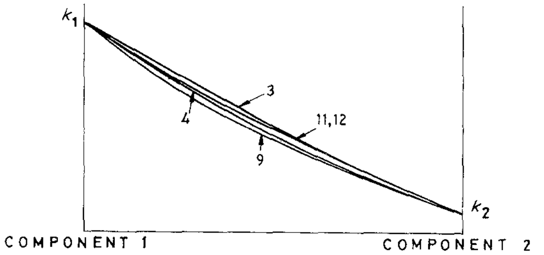


Figure 6 Conductivity curves calculated from Equations 3, 4, 9, 11 and 12 for a binary system where $k_1 = 2k_2$.

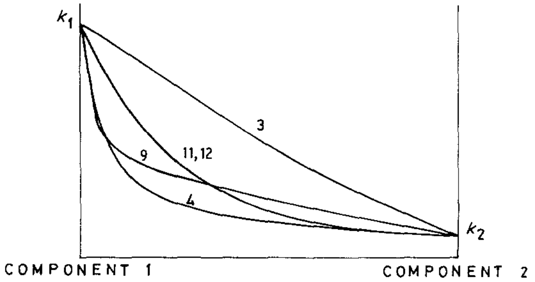


Figure 7 Conductivity curves calculated from Equations 3, 4, 9, 11 and 12 for a binary system where $k_1 = 50k_2$.

It is evident that not all the Equations represented in Fig. 7 can reflect the true binary composite conductivity. A final selection was therefore made after comparison of the conductivities calculated for a real system with the experimentally-measured conductivities.

The system chosen for this comparison was a porous, debased alumina.* Samples were made using a pressing and firing method, and a range of porosities was achieved by incorporation of acrylic powder which burned out during firing. Porosities

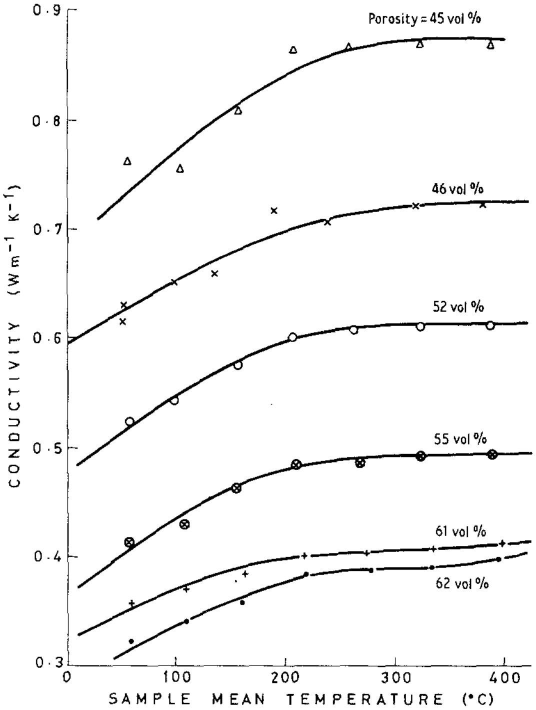


Figure 8 The effective thermal conductivity of porous debased alumina as a function of mean sample temperature.

---

*Morgan Δ-grade.

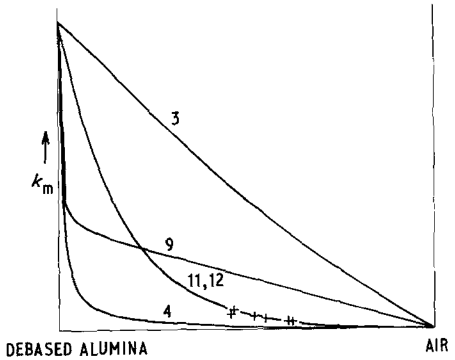


Figure 9 Conductivity curves calculated from Equations 3, 4, 9, 11 and 12, and experimental values for the debased alumina–air system.

and pore-size distributions were subsequently determined by mercury porosimetry. It was found that greater than 90% of the pores were in the size range 10 to 50 μm, that is, smaller than the convection criterion suggested by McAdams [30] and greater than the sub-micrometre size which gives rise to gas conduction anomalies [31]. The effective thermal conductivities of the fired samples were measured for a range of sample mean temperatures, using a Foseco KU-meter, and the results are shown in Fig. 8.

The effective thermal conductivities of these samples for a mean temperature, $\bar{T}$, of 100° C were then compared with the predictions of Equations 3, 4, 9, 11 and 12 using 100° C conductivity data for dense debased alumina [32] and air [33]. Fig. 9 shows the curves predicted by these equations, and the experimental results.

The geometric mean equations, Equations 11 and 12, give predicted lines which pass through the experimental data, suggesting that these equations are the most suitable as a basis from which to develop a computer program for prediction of the thermal conductivities of multicomponent systems.

As a further check on the applicability of the geometric mean equations to real systems, the thermal conductivities of various materials were calculated from their known compositions and the conductivities of their components, obtained from the literature [33], using Equation 11, and compared with measured conductivities. Direct measurement of conductivities at 20° C, which was the temperature for which literature values were usually quoted, was not possible with the Foseco instrument because it was designed for high-temperature use, and was inaccurate at temperatures less than 60° C. However, it was found that, between 60 and about 300° C there was a straight-line relationship between conductivity and temperature, and the line could be extrapolated to give satisfactory 20° C values. Table II compares the predicted and measured conductivities, and in most cases there is good agreement, which gives further support to the use of the geometric mean equations as a basis for computer predictions.

As can be seen from Table II, the geometric mean equation, Equation 11, gives predicted conductivities which are in very good agreement with measured conductivities. This agreement is found even when the conditions given in Section 2

TABLE II Comparison of the measured and predicted thermal conductivities of various multicomponent materials

Material	Number of components	Porosity (vol %)	Temperature (° C)	Conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)	
				Experimental value	Predicted value
Debased alumina	2	45	100	0.75	0.81
Debased alumina	2	46	100	0.66	0.80
Debased alumina	2	52	100	0.55	0.57
Debased alumina	2	55	100	0.44	0.47
Debased alumina	2	61	100	0.36	0.33
Debased alumina	2	62	100	0.33	0.31
Cenolite* insulation	3	88	20	0.13	0.13
Polystyrene bead–cement–PFA	4	55	20	0.18	0.18
PFA cenosphere–polymer	3	88	20	0.12	0.12
Sodium silicate foam	2	97	20	0.04	0.04
Glass fibre–Al foil	3	97	20	0.04	0.04
PFA cenospheres, unbonded	2	90	20	0.11	0.11
FPA cenosphere-filled sodium silicate foam	3	94	20	0.09	0.08

*Cenolite is a fire protection and thermal insulation material, based on pulverized fuel ash cenospheres [34].

are not strictly met, for example, in the cases of the glass fibre–aluminium foil mat and the polystyrene bead–cement composite. In the first case the agreement can be attributed to the high volume fraction of porosity, which dominates the effective conductivity both in experiment and prediction. In the latter case, however, it must be assumed that, in spite of the relatively large size of the beads, they are still small enough, when randomly arranged, to satisfy the conditions. Therefore, Equation 11 appears to be appropriate for a variety of insulation materials with porosities ranging from 50 to 97 vol%, which can contain phases with dimensions up to 20% of the sample size, as long as there is random phase distribution. This gives confidence in the reliability of conductivity predictions for a wide range of insulation materials.

4. Computer prediction of thermal conductivities

4.1. Program development

Using Equation 11 the conductivity of a material after manufacture can be calculated from the known conductivities of its constituents and their volume fractions, determined from their weights and densities and the measured volume of the product. Although such employment of the equation for multicomponent materials may be useful, it is limited, and the aim of the present work is to develop a computer program with more scope, as an aid to the design and development of materials. It is required that conductivity data for the whole compositional spectrum of the system of interest should be able to be displayed, and this may be achieved conveniently by using ternary diagrams similar to those used in phase equilibrium

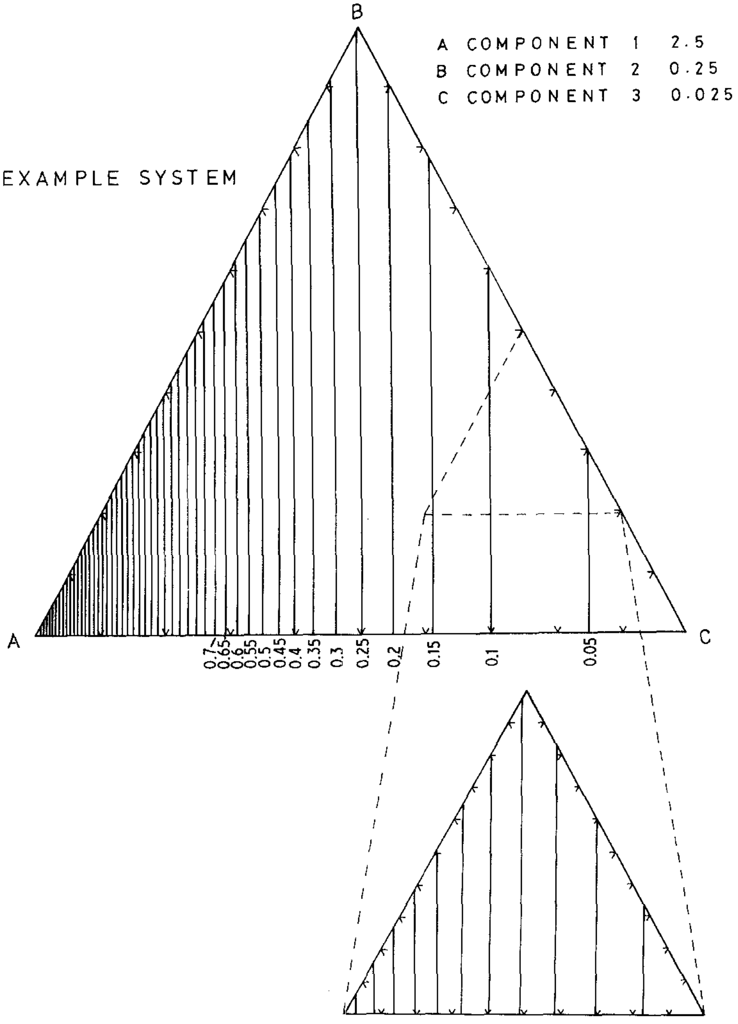


Figure 10 Full and selected-area ternary diagrams for a hypothetical three-component system.

studies. By calculating conductivities for the whole range of compositions, an iso-conductivity (iso-k) line diagram can be constructed, from which the effects of composition on conductivity can readily be seen. Use of the geometric mean equation gives straight and parallel iso-k lines, and this result has allowed the program to be developed to provide, not only data, but a ternary diagram, using the graphics facility of a Tektronix 4051 microprocessor. Program sub-routines have been developed to allow the computer to

(a) deal with systems containing up to six components by arranging them into three groups, so that conductivity predictions can still be presented in the form of a ternary diagram and

(b) magnify selected portions of a diagram and improve the reading accuracy by giving iso-k lines at smaller intervals. Fig. 10 shows full and selected-area diagrams for a hypothetical three-component system.

4.2. Program application

As a materials design tool the computer program is used firstly to give the full ternary, iso-conductivity diagram for the system to be studied. From this initial diagram a composition field can then be chosen to give a material with the required conductivity, and a selected area from the field magnified. When suitable compositions have been found, appropriate fabrication methods can be selected. The program has been used in this manner for the development of thermal insulation materials based on pulverised fuel ash [35]. A copy of the program listing can be found in [35].

5. Discussion

The published equations examined in this work all suffer from the need to simplify the conductivity problem from one in which heat is transferred by a complex combination of conduction, convection and radiation mechanisms to one in which conduction only is considered. Strictly this limits their application to situations in which the non-conduction components are negligible in comparison with the conduction component.

Both radiative and convective heat transfer are dependent on factors such as geometry, size and temperature gradient across pores. These factors are controlled by the structure of the material and the thermal conditions of the working environment and, as such, are difficult to incorporate successfully in a predictive expression. Radiative heat transfer exhibits a cubic relationship with the absolute temperature of the material [31] and the equations derived from the radiant conductivity, k_r, are of the form

$$k_r = F\sigma T^3, \tag{18}$$

where F is a function of the component materials and their geometric configuration, σ is the Stefan–Boltzmann constant and T is the temperature (K).

A more specific equation is given by Kingery [36] for the radiative conductivity of a small spherical pore

$$k_r = \frac{(T_1^4 - T_2^4)e\sigma r^2 d_p}{T_1 - T_2}, \tag{19}$$

where T_1 and T_2 are the surface temperatures of the pore (K), e is the emissivity of the surface, r is the refractive index of the solid and d_p is the pore diameter.

Equation 19, although it relates to a specific situation of temperature gradient and pore size, is inadequate as a description of a real porous insulation in which there will be a size distribution of non-spherical pores and there is the likelihood of some radiative heat transfer through the solid phase.

From this equation, however, we can see that for small pores the radiative heat transfer will be small, due both to the value of d_p and also to the smaller temperature difference, $\Delta T, (T_2 - T_1 = \Delta T)$ across the pore. If, using Equation 19, the hypothetical cases are considered of spherical, air-filled pores of various sizes in an insulation material such that the temperature gradient is $200\,\text{K cm}^{-1}$ then radiation across the pore will contribute 20% of the heat transfer at 400 K for a 100 μm diameter pore, 700 K for a 50 μm diameter pore and 3000 K for a 5 μm diameter pore. The majority of insulation materials have pores in this range, and thus from a radiation heat transfer point of view the failure to include this mechanism up to these limits will not give rise to major errors.

Convection within the cavities of a porous body has been discussed theoretically by McAdams [30] who set down criteria of pore size and temperature gradient for onset of convection. These are supported by empirical data [37], which suggest that ordinarily the convective contribution to the thermal conductivity is small in comparison with the contributions of conduction and radiation. Indeed Gorring and Churchill [31] state that no great error occurs if convection is neglected in insulation materials.

The preceding discussion has suggested that serious limitations exist for all the equations examined in this work. If, however, consideration is restricted to materials in which the pore size and the temperature differential across the pores are small then only small errors will result. These errors will be further minimized by the use of experimentally-obtained, effective thermal conductivities which must incorporate all modes of heat transfer at the selected temperature.

The aim of this work was to select an expression from those proposed in the literature which gives demonstrably good agreement with experimental multi-component thermal conductivities: it has been shown that the geometric mean equation, Equation 11, is the most satisfactory.

6. Conclusions

The flux-law, Ohm's law and empirical equations proposed in the literature for the prediction of the thermal conductivities of multicomponent materials have been examined, and their validity tested.

Theoretical and experimental evaluation of the expressions has shown that a geometric mean equation, Equation 11, is the most suitable for use as the basis of a computer program to predict the thermal conductivities of multicomponent systems.

When used in conjunction with a microprocessor and graphics facility, the computer program can provide ternary iso-conductivity diagrams of two kinds. The first, simpler option, uses the conductivities of three components (or groups of components) to produce a full composition–conductivity diagram. The second option magnifies selected areas of the diagram and shows smaller conductivity intervals.

Use of the geometric mean equation, Equation 11, limits the validity of predictions to materials formed by a physical mixing of known phases, of small size in relation to that of the sample, and randomly mixed. Conductivity data must be available for the components in the form in which they appear in the composite material.

The program was used to predict the conductivities of a variety of materials, and it was found that, when the above conditions were fulfilled, there was good agreement with measured conductivities.

References

1. J. C. MAXWELL, "Electricity and Magnetism" Vol. 1 (Clarendon Press, Oxford, 1904).
2. D. A. DE VRIES, *Soil Sci.* **73** (1952) 83.
3. A. A. NEKRASOV, *Zh. Tech. Fiz.* **10** (1940) 2.
4. V. Z. BOGOMOLOV, *Trans. Phys. Astron. Inst.* **3** (1941).
5. A. MATSUURA, T. AKEHATA and T. SHIRAI, *Heat Trans. Jap. Res.* **4** (1975) 79.
6. G. K. BATCHELOR and R. W. O'BRIEN, *Proc. R. Soc.* **A355** (1977) 313.
7. D. KUNII and J. M. SMITH, *A.I.Ch.E. J.* **6** (1960) 71.
8. A. J. TAMARIN, *Inzh.-Fiz. Zh.* **18** (1970) 566.
9. G. N. DUL'NEV and Z. V. SIGALOVA, *Int. Chem. Engng.* **5** (1965) 218.
10. H. W. RUSSELL, *J. Amer. Ceram. Soc.* **18** (1935).
11. M. R. J. WYLLIE and P. S. SOUTHWICK, *J. Petrol. Tech.* **6** (1954) 44.
12. W. WOODSIDE and J. H. MESSMER, *J. Appl. Phys.* **32** (1961) 1688.
13. E. H. RATCLIFFE, *J. Appl. Chem.* **18** (1968) 25.
14. G. N. DUL'NEV and YU. P. ZARICHNYAK, *Heat Trans. Sov. Res.* **2** (1970) 89.
15. V. I. ODELEVSKIY, *Zh. T. F.* **21** (1951) 167.
16. G. N. DUL'NEV, *I. F. Zh.* **9** (1965) 4.
17. P. N. LAL CHAURASIA, D. R. CHAUDHARY and R. C. BHANDARI, *J. Appl. Chem. Biotechn.* **24** (1974) 437.
18. D. R. CHAUDHARY and R. C. BHANDARI, *Ind. J. Pure Appl. Phys.* **5** (1967) 415.
19. D. R. CHAUDHARY and R. C. BHANDARI, *ibid.* **6** (1968) 274.
20. D. R. CHAUDHARY and R. C. BHANDARI, *Brit., J. Appl. Phys.* **2** (1969) 609.
21. R. A. CRANE and R. I. VACHON, *Int. J. Heat Mass Trans.* **20** (1977) 711.
22. M. FUJITSU, M. HASATANI and S. SUGIYAMA, *J. Chem. Eng. Jap.* **10** (1977) 242.
23. A. MISSENARD, "Le conductivité thermique des solides, liquides, gas et leur melanges" (Éditions Eyrolles, Paris, 1965) p. 336.
24. K. LICHTENECKER, *Z. Physik* **27** (1926) 115.
25. Y. ASSAD, Ph.D. thesis, University of California (1955).
26. A. SUGAWARA and Y. YOSHIZAWA, *Aust. J. Phys.* **14** (1961) 469.
27. J. C. CHENG and R. I. VACHON, *Int. J. Ht. Mass Transfer* **12** (1969) 249.
28. J. SCHUHMEISTER, *Ber. K. Akad. Wein* **76** (1877) 283.
29. J. M. WIMMER, H. C. GRAHAM and N. M. TALLAN, "Electrical Conductivity in Ceramics and Glasses, Vol. 4, edited by Tallan (Marcel-Dekker, New York, 1974) p. 625.
30. W. H. MCADAMS, "Heat Transmission" (McGraw-Hill Book Co., New York, 1951) p. 19.
31. R. L. GORRING and S. W. CHURCHILL, *Chem. Eng. Prog.* **57** (1961) 53.
32. A. R. L. MOFFAT, in "Special Ceramics" edited by P. Popper (Academic Press, New York, 1965) p. 213.
33. "American Institute of Physics Handbook", 3rd edn

edited by M. W. Zemansky (McGraw-Hill Book Co., New York, 1972).
34. A. BRIGGS, Proceedings of the 4th Cimtech Conference on Energy and Ceramics, St. Vincent, Italy, May 1979.
35. D. M. DAWSON, M.Sc. thesis, University of Bath (1980).
36. W. D. KINGERY, "Introduction to Ceramics", 2nd edn (Wiley-Interscience, New York, 1976) 634.
37. J. D. VERSCHOOR and P. GREEBLER, *Trans. ASME* 74 (1952) 961.

Received 10 April and accepted 15 May 1981.